Three-Dimensional, Three-Phase Discrete-Fracture Reservoir Simulator Based on Control Volume Finite Element (CVFE) Formulation

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Outline of this presentation

- Control volume finite element formulation
- Case studies
- Summary

Current state of reservoir simulation

- Finite-difference simulators dominate the market
- Very difficult to simulate complex domains with complicated well patterns, and domains with arbitrary fracture patterns
- Finite-element models with body-fitting meshes more appropriate in simulating these systems

Control Volume Formulation

Three-phase flow in porous media

$$-\nabla \cdot \underline{u}_0 = \frac{\partial}{\partial t}\left(\frac{1}{B_o}\phi S_o\right) + q_o$$

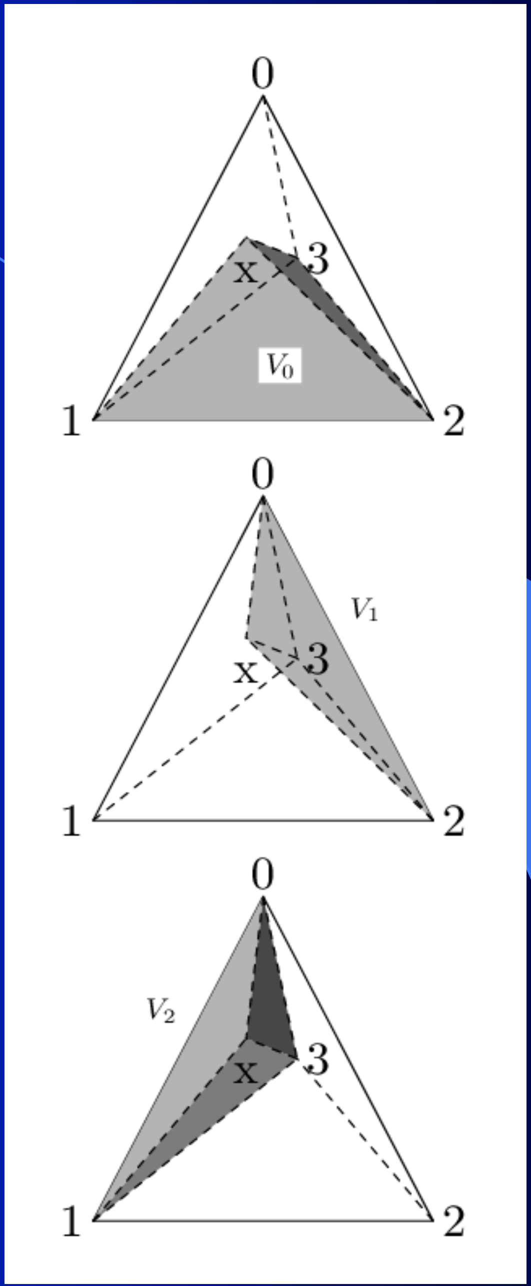
$$-\nabla \cdot \underline{u}_w = \frac{\partial}{\partial t}\left(\frac{1}{B_w}\phi S_w\right) + q_w$$

$$-\nabla \cdot \left(R_s \underline{u}_o + \underline{u}_g\right) = \frac{\partial}{\partial t}\left(\frac{R_s}{B_o}\phi S_o + \phi \frac{S_g}{B_g}\right) + R_s q_0 + q_{fg}$$

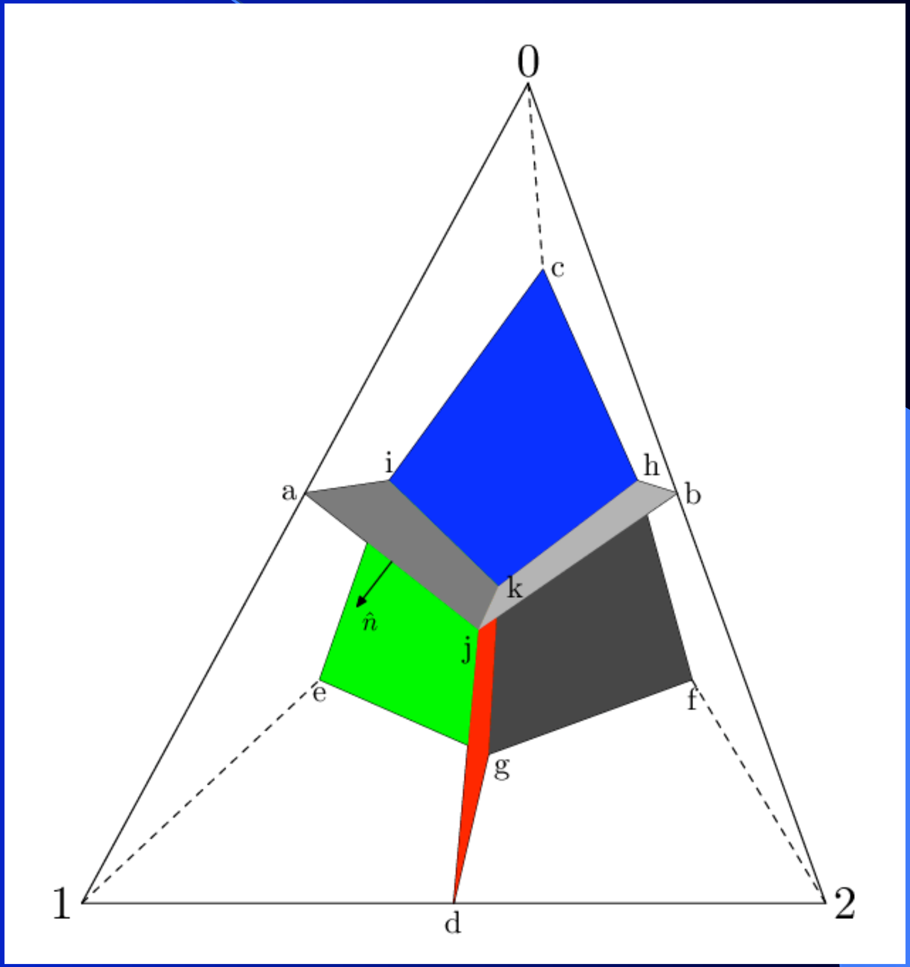
The lowest order interpolation function from the Lagrange family – the linear interpolation functions are used.

The position of any given point x in a tetrahedral element can be established uniquely by the volumes enclosed by the vertices of a tetrahedron.

$$X = (L_0, L_1, L_2) = \left(\frac{V_0}{V}, \frac{V_1}{V}, \frac{V_2}{V} \right)$$



A tetrahedral element with associated control volumes



In the control-volume formulation, the fluid potential and saturation values are defined on the vertices of the tetrahedron.

The fluid potential value in the tetrahedron is interpolated using the interpolation functions. The fluid saturation value is calculated for each control volume from the solution of the residual equations.

Residual function for node 0 (u is the phase velocity, phase subscripts dropped)

$$F_0 = \int_{CV_0} \nabla \cdot u + \frac{\partial C_0}{\partial t} dX = \int_{aikj} \underline{u} \cdot \hat{n} ds + \int_{bjkh} \underline{u} \cdot \hat{n} ds + \int_{chki} \underline{u} \cdot \hat{n} ds + \int_{CV_0} \frac{\partial C_0}{\partial t} dX$$

For the oil phase,

$$C_0 = \frac{1}{B_o} \phi S_o$$

The velocity for each of the phases is obtained using the Darcy's law.

$$\underline{u} = -\frac{k_r k}{B\mu}\left(\nabla P\right)$$

To account for gravity, the term in parenthesis of the above equation is changed to $\left(\nabla P + g\rho\nabla Z\right)$.

The first component of the flux is expanded as follows.

$$f_{0,aikj} = \int_{aikj} \underline{u} \cdot \hat{n} ds = \int_{aikj} \left(u_x n_x + u_y n_y + u_z n_z\right) ds$$

After substituting and expanding the geometric terms, the final form of the flux equation is:

$$f_{0,aikj} = \frac{1}{12}\frac{k_{r01}}{B_{01}\mu_{01}}\left\{\begin{array}{l}\left(k_{xx}\frac{\partial P}{\partial x} + k_{xy}\frac{\partial P}{\partial y} + k_{xz}\frac{\partial P}{\partial z}\right)\\ \left[(y_2 - y_a)(z_3 - z_a) - (y_3 - y_a)(z_2 - z_a)\right]\\ + \left(k_{yx}\frac{\partial P}{\partial x} + k_{yy}\frac{\partial P}{\partial y} + k_{yz}\frac{\partial P}{\partial z}\right)\\ \left[(x_3 - x_a)(z_2 - z_a) - (x_2 - x_a)(z_3 - z_a)\right]\\ \left(k_{zx}\frac{\partial P}{\partial x} + k_{zy}\frac{\partial P}{\partial y} + k_{zz}\frac{\partial P}{\partial z}\right)\\ \left[(x_2 - x_a)(y_3 - y_a) - (x_3 - x_a)(y_2 - y_a)\right]\end{array}\right\}$$

Upstream weighting for relative permeability.

- For $f_{0,aikj} > 0,\ k_{r01} = k_{r0} = k_r(S_0)$
- For $f_{0,aikj} < 0,\ k_{r01} = k_{r1} = k_r(S_1)$

The final form of the residual function at node 0:

$$F_0^{n+1} = f_{0,aikj}^{n+1} + f_{0,bjkh}^{n+1} + f_{0,chki}^{n+1} + \frac{V}{4 \cdot \Delta t}\left(C_0^{n+1} - C_0^{n}\right)$$

Computations

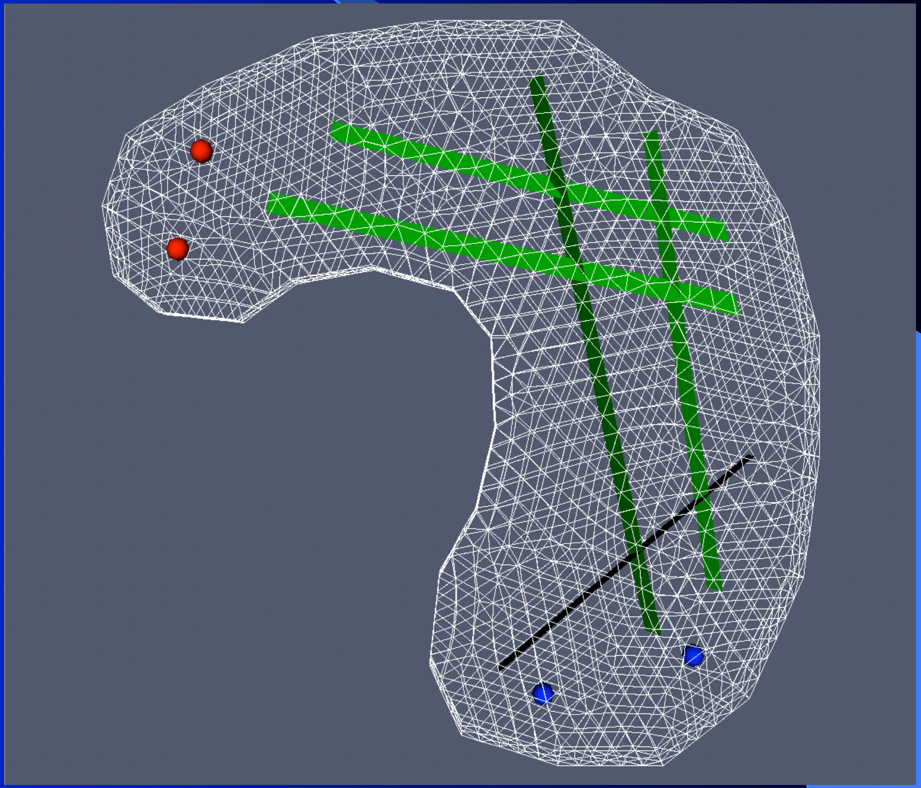
- Fully implicit mode by Newton’s method
- Linear solvers provided by PETSc
- Written in C++
- Modular and parallel

Three-dimensional, two-phase simulations

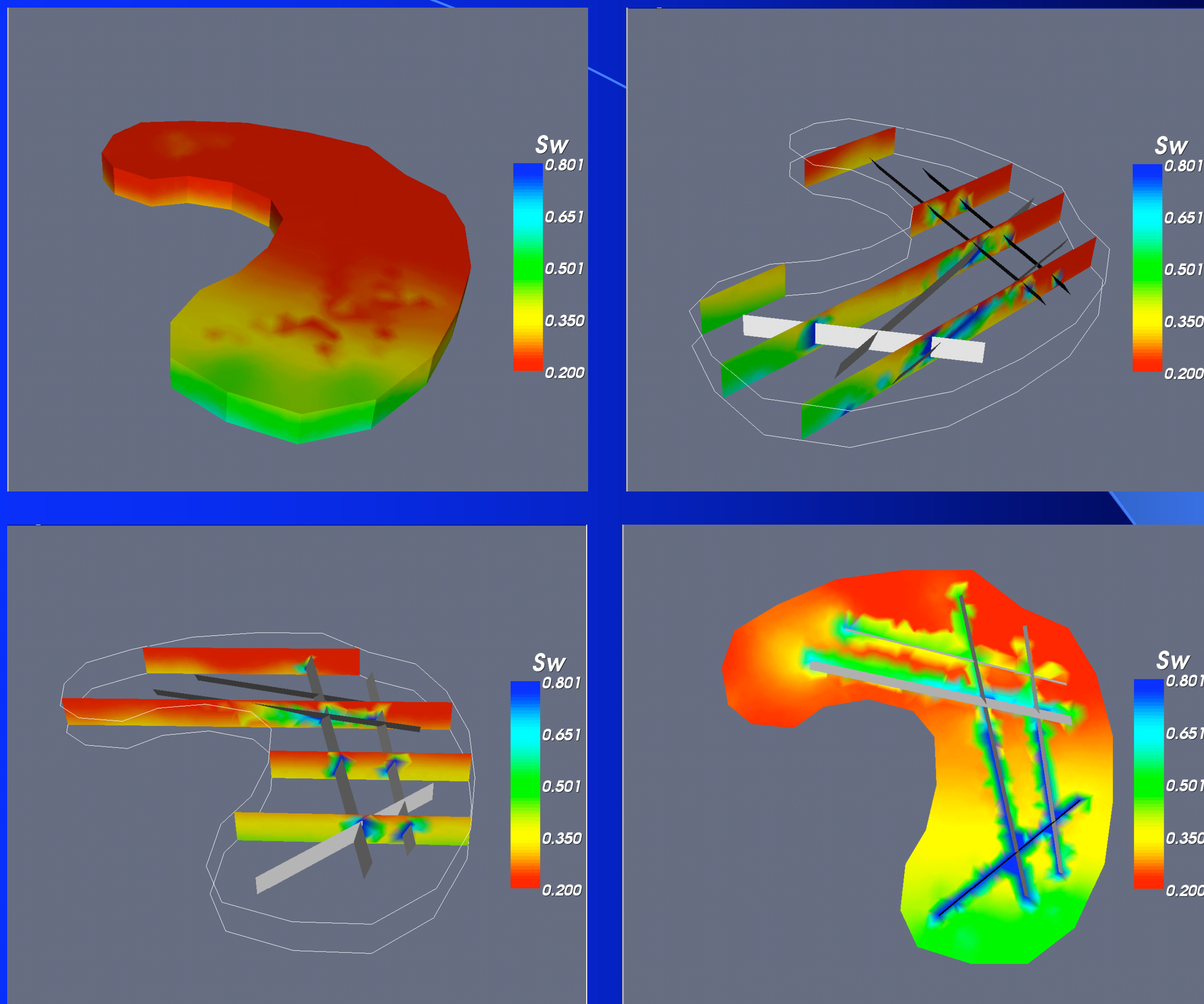
Complicated boundary

Domain with fractures

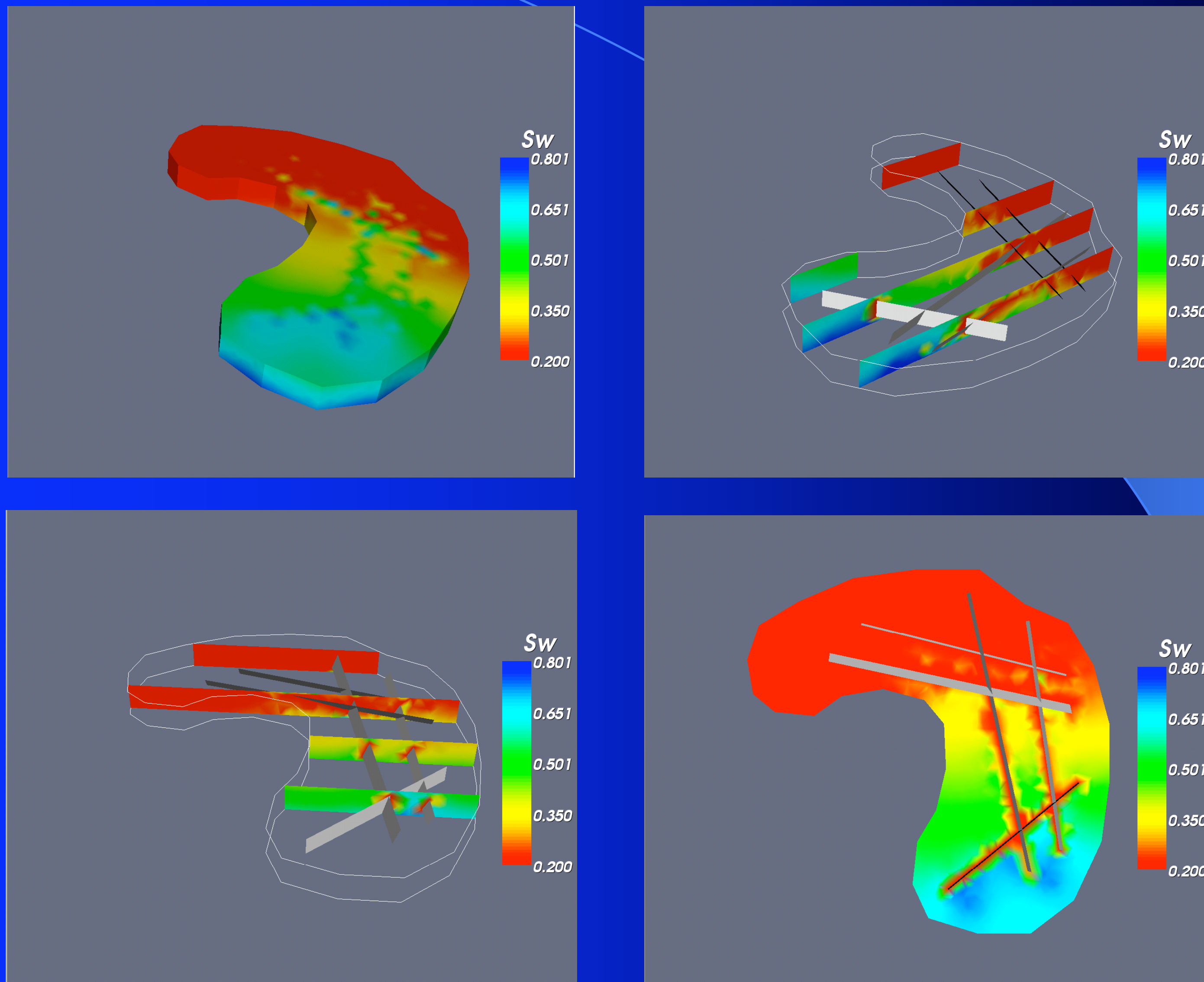
Difficult to simulate with existing simulators



Negative capillary pressure



Positive capillary pressure



The figure shows water saturations; when the rock is “water-wet” or imbibes water, oil is swept more effectively.

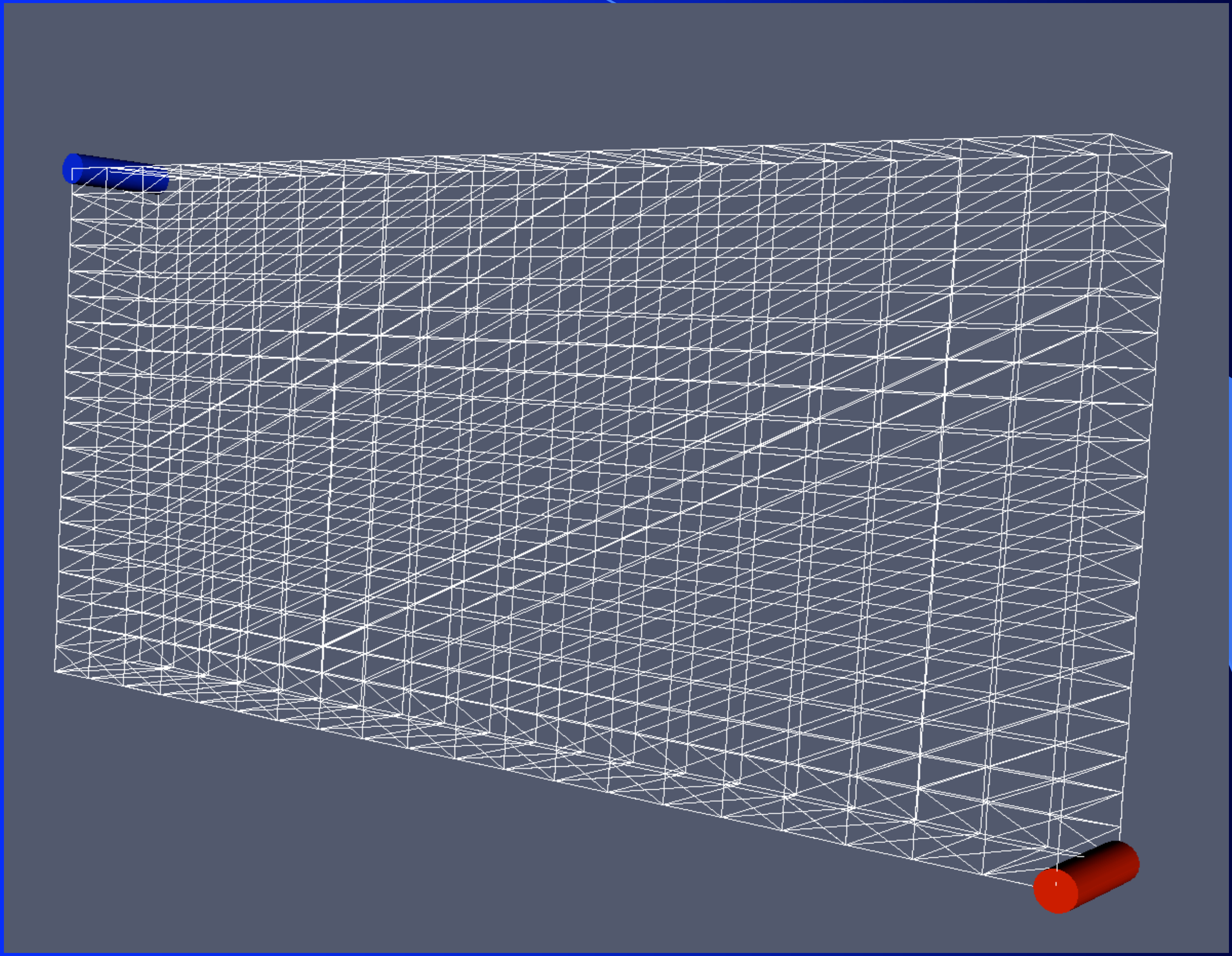
Water channeling and segregation occurs along high permeability fractures.

These type of studies are difficult to perform using the finite-difference simulators, since the fractured domains are difficult to represent.

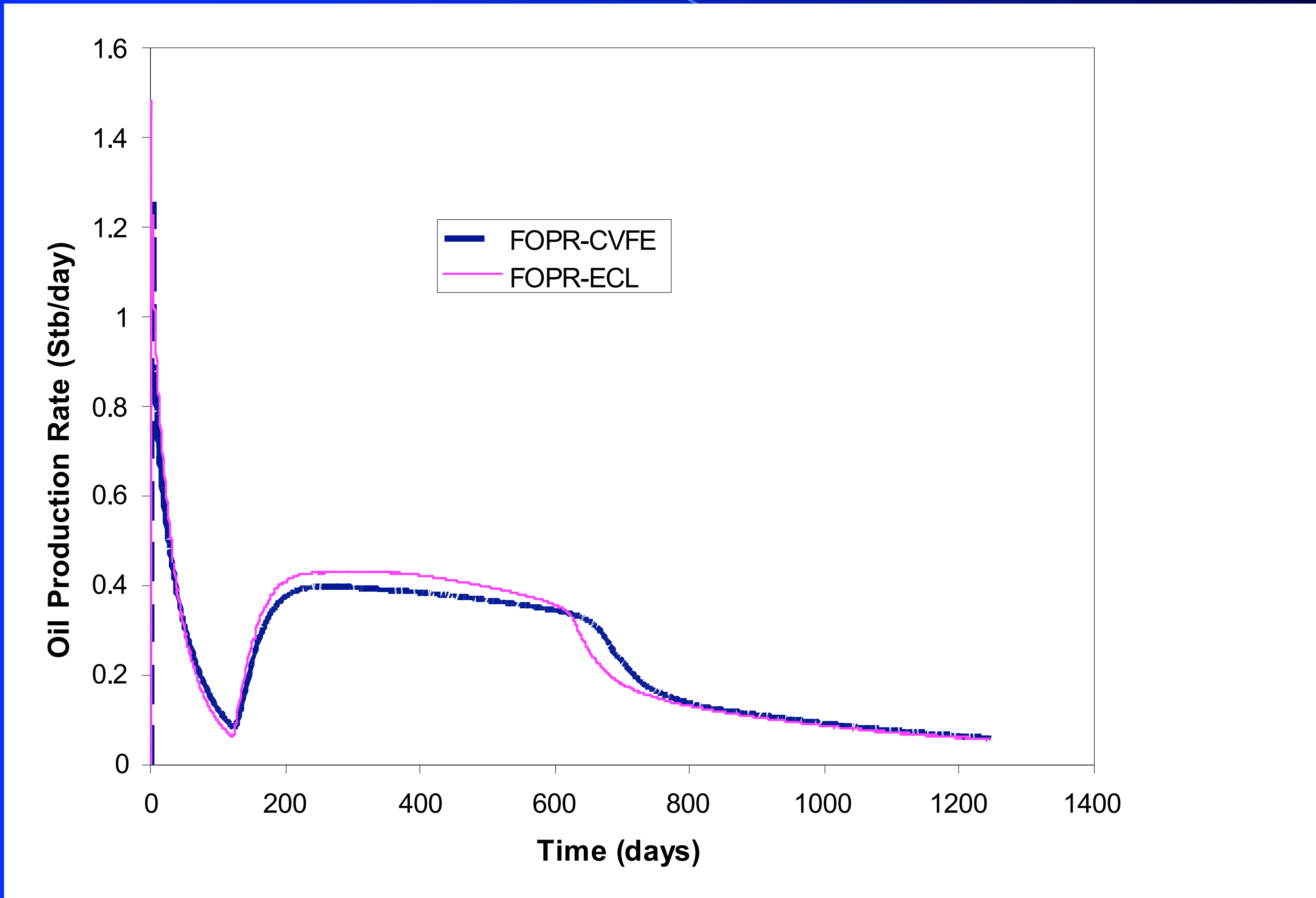
Three-dimensional, three-phase simulations

Domain	10 feet x 100 feet x 50 feet
CVFE simulations: Number of tetrahedrons	2400
Eclipse simulations: Number of blocks	400
Horizontal injector	At the top, (shown in blue)
Horizontal producer	At the bottom, (shown in red)
Initial reservoir pressure	3300 psia
Initial bubble point pressure	3200 psia
Production bottom hole pressure	2900 psia
Primary production for	120 days
Water injection pressure	3300 psia
Water injection continued until	A water cut of 80% obtained

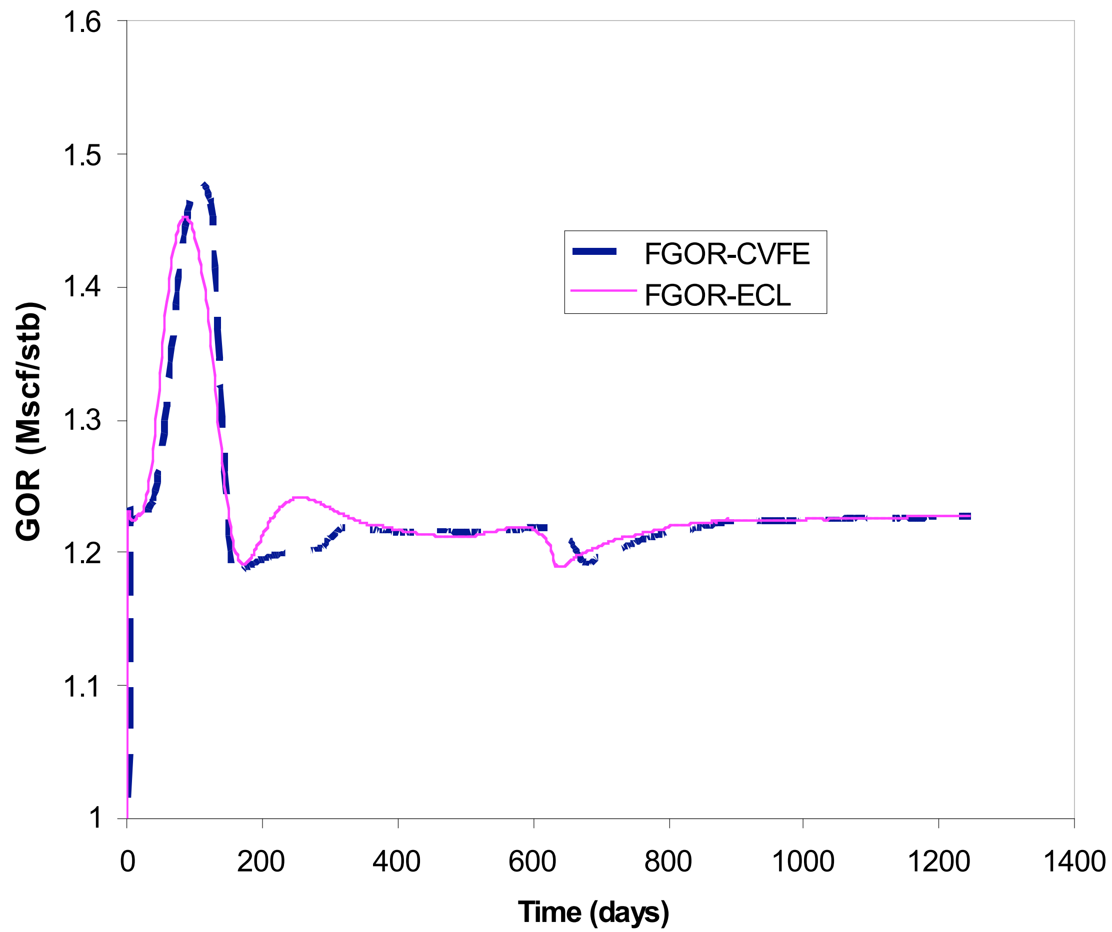
The regular domain using a tetrahedral mesh



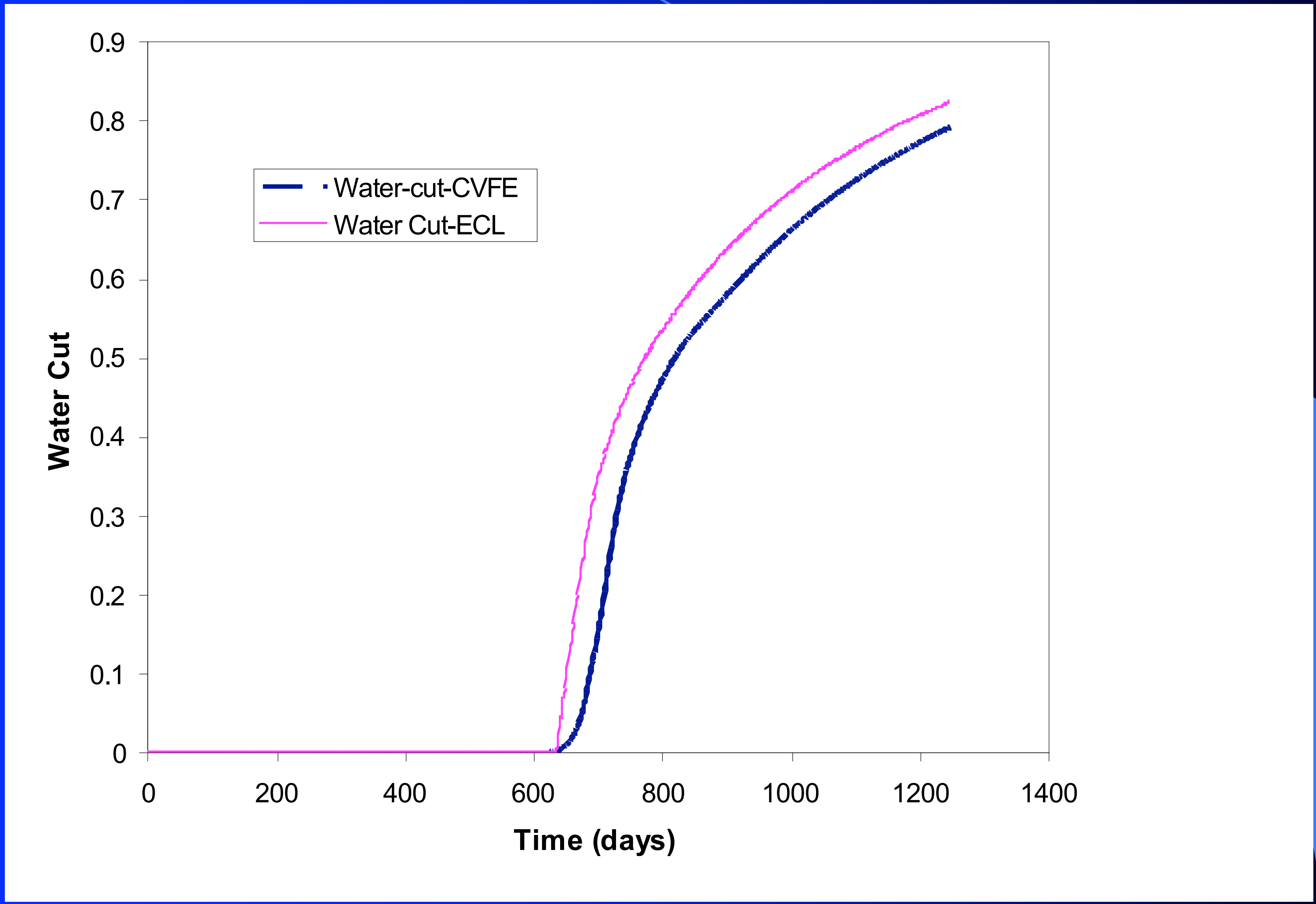
Oil Production Rate



Gas-Oil Ratio



Water Cut

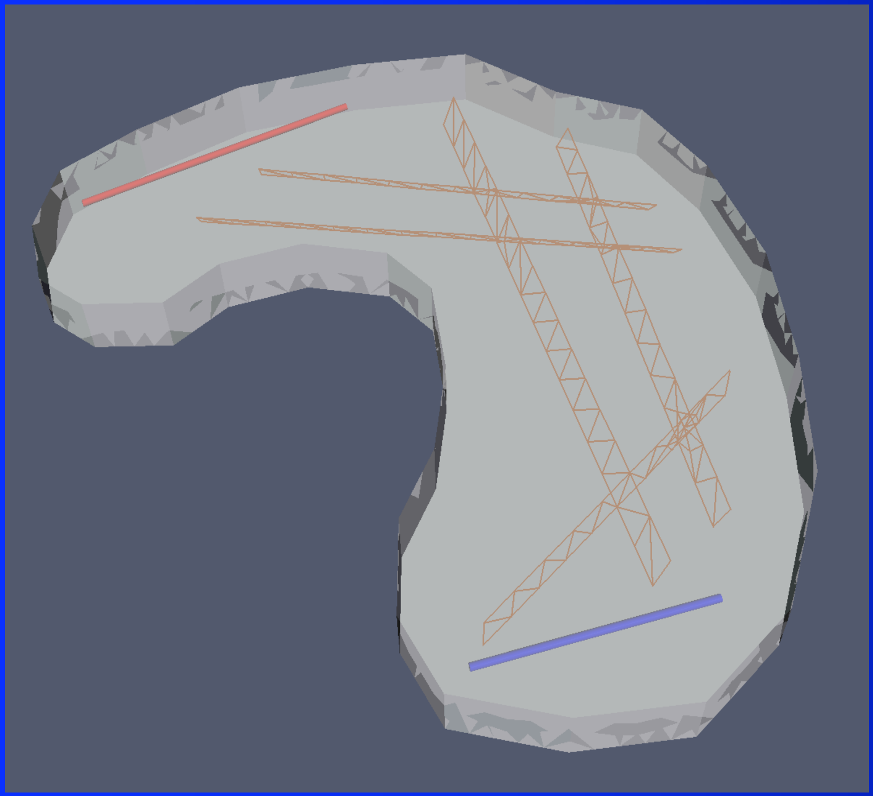


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Water Cut
0
200
400
600
800
1000
1200
1400
Time (days)
Water-cut-CVFE
Water Cut-ECL

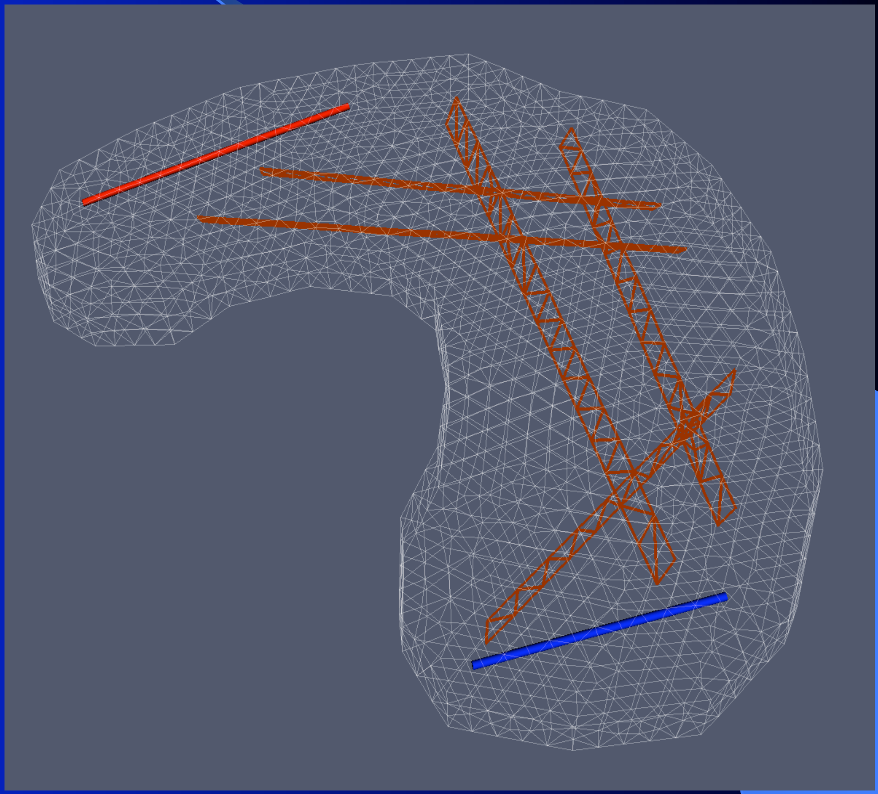
Three-dimensional, three-phase simulations with a complex domain

- Domain same as the three-dimensional, two-phase simulations
- Horizontal wells, injector shown in blue, producer shown in red
- Partially penetrating and tilted fractures

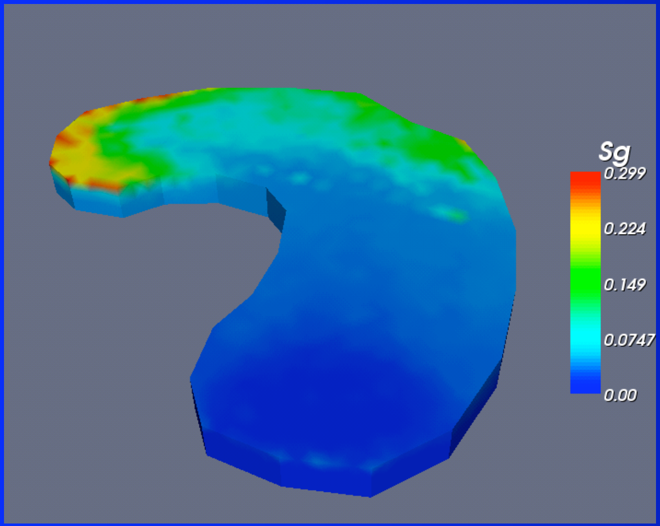
The three-dimensional domain

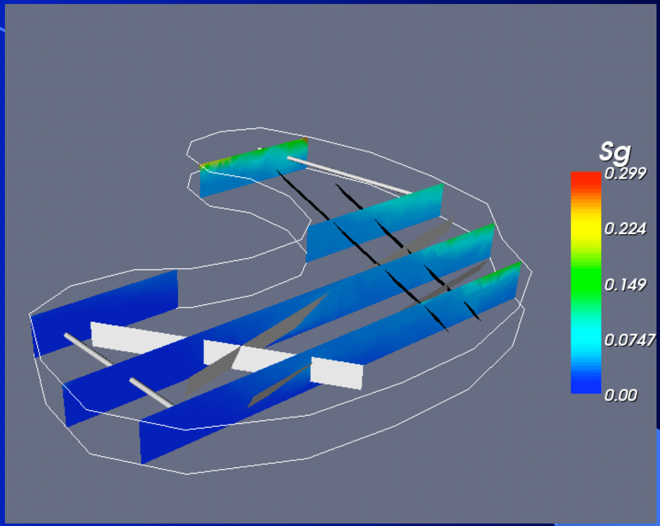


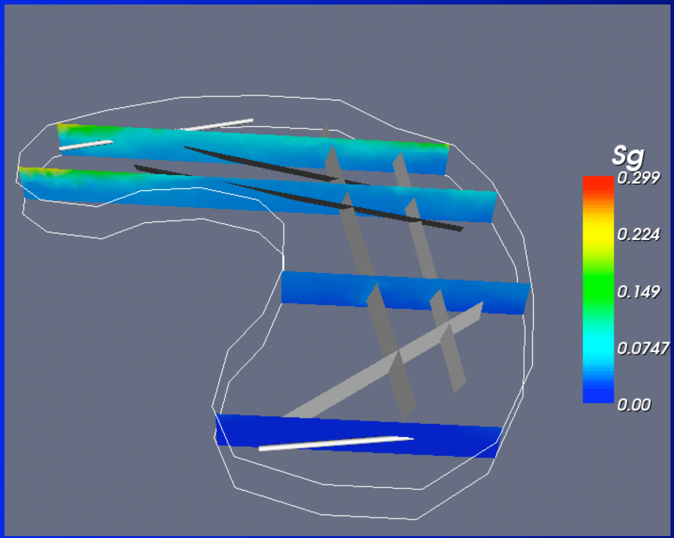
The tetrahedral mesh



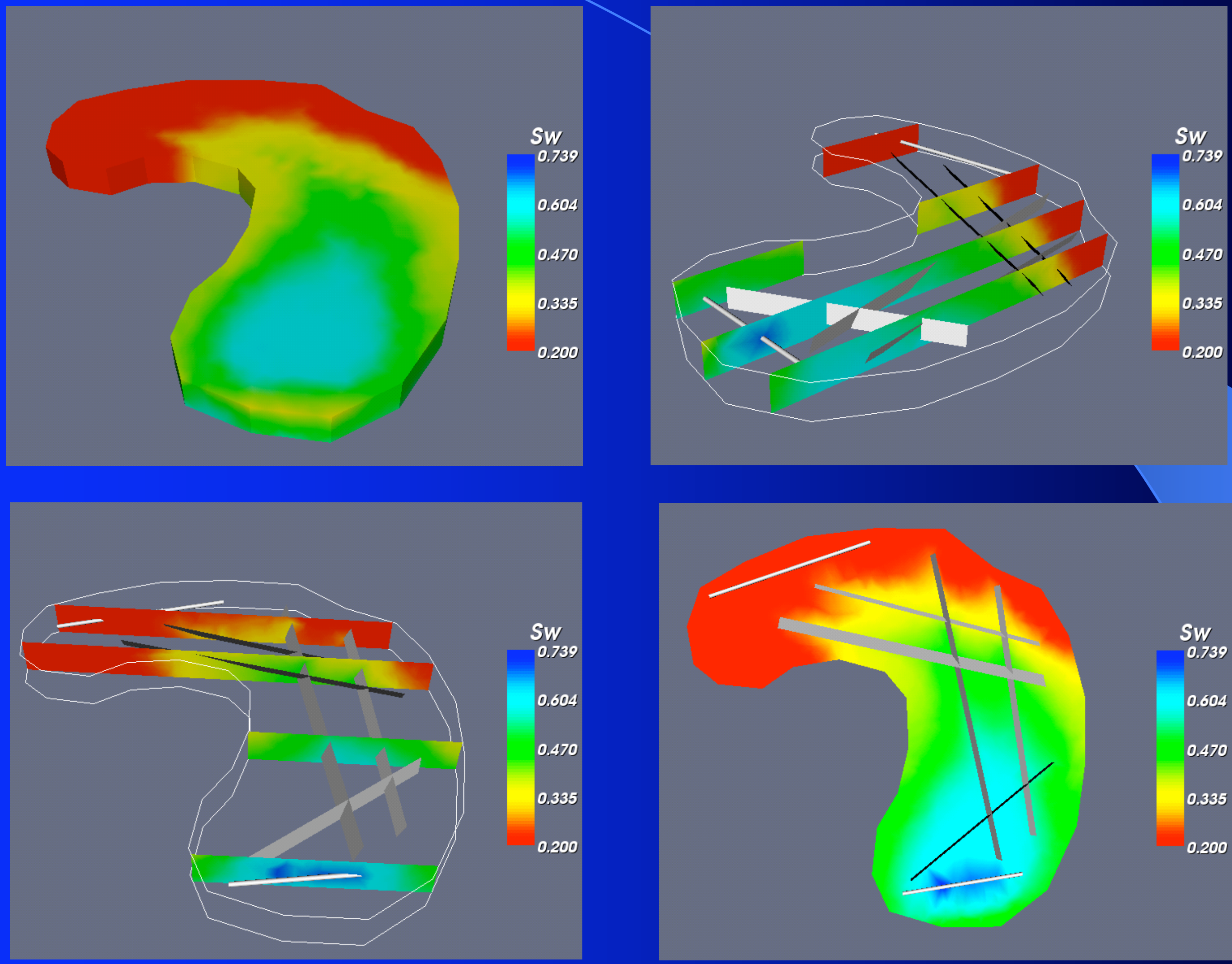
Gas saturations at 953 days





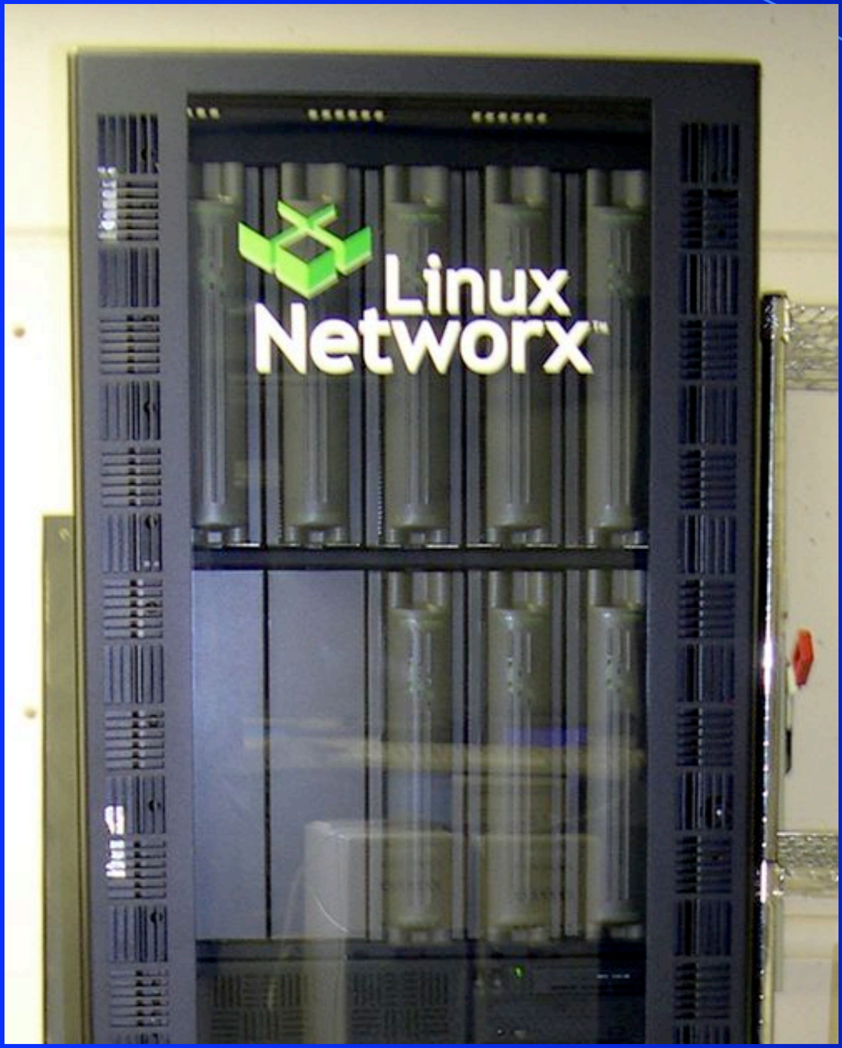


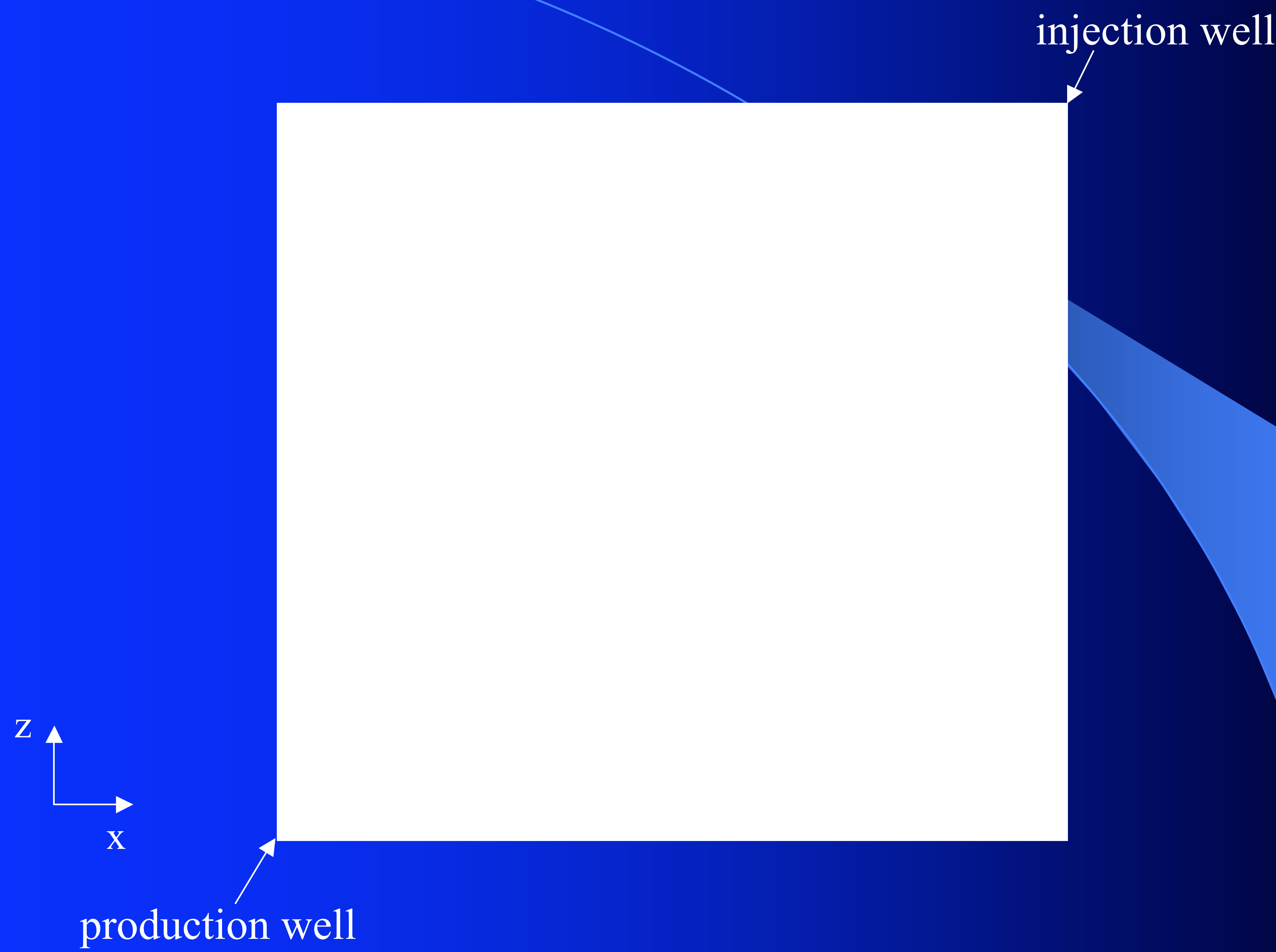
Water saturations at 3431 days, some channeling in the Fractures, but good sweep

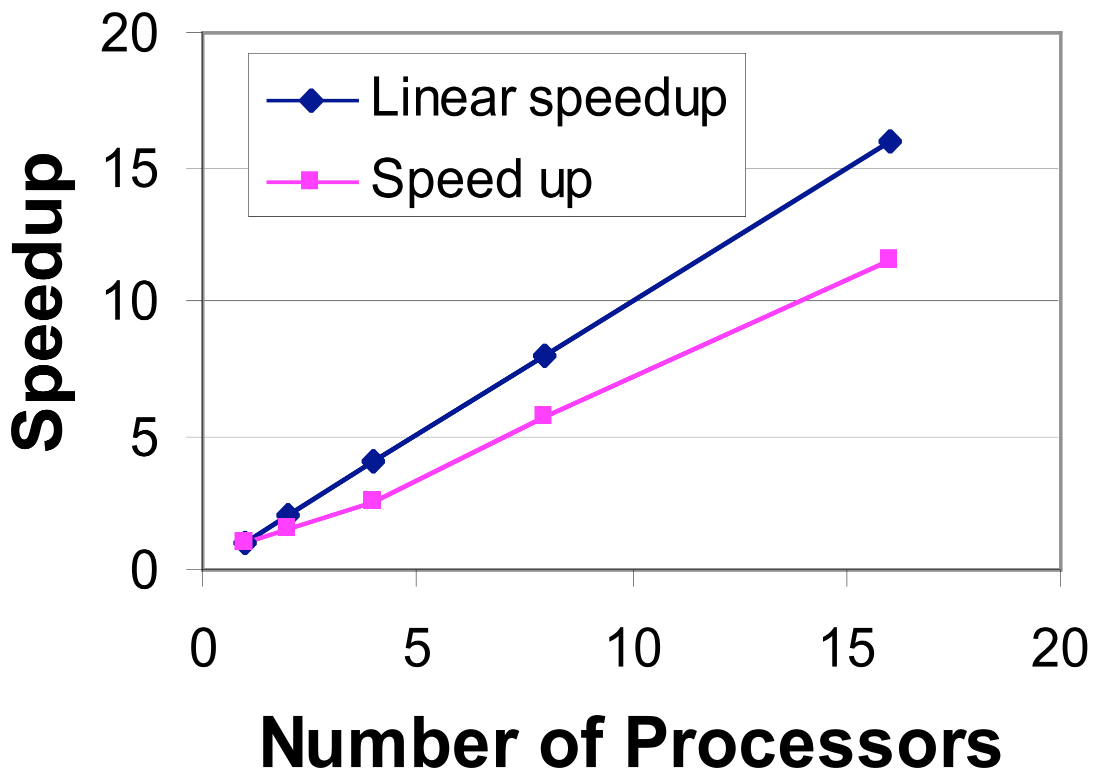


Parallel performance

- Linux cluster – 64-bit, 18 processors
- Two dimensional, two-phase simulation
- 250,000 nodes
- MPI-based code
- Good scalability
- Speedup of 12 on 16 processors


Linux
Networx



injection well
z
x
production well



Linear speedup
Speed up
Speedup
Number of Processors
0
5
10
15
20

Summary

– A new three-dimensional, three-phase black-oil simulator based on the control-volume finite-element formulation is developed.
– Results from the simulator are in good agreement with results from Eclipse.
– Other case studies, with tilted, partially penetrating faults, which would be difficult to represent in finite-difference formulations are presented to demonstrate the applicability of the simulator.

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Questions?